Generating Random Logic Programs Using Constraint Programming

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FATA Seminar

How Many Programs Are Used to Test Algorithms?

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Anytime Inference in Probabilistic Logic Programs with $T_{\mathcal{P}}$-Compilation

4

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Inference and learning in probabilistic logic programs using weighted Boolean formulas

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***k*-Optimal: a novel approximate inference algorithm for ProbLog**

Joris Renkens · Guy Van den Broeck · Siegfried Nijssen

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On the Efficient Execution of ProbLog Programs

Angelika Kimmig[1], Vítor Santos Costa[2], Ricardo Rocha[2], Bart Demoen[1], and Luc De Raedt[1]

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On the Implementation of the Probabilistic Logic Programming Language ProbLog

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1

ProbLog Technology for Inference in a Probabilistic First Order Logic

Maurice Bruynooghe and **Theofrastos Mantadelis** and **Angelika Kimmig** and **Bernd Gutmann** and **Joost Vennekens** and **Gerda Janssens** and **Luc De Raedt**[1]

2

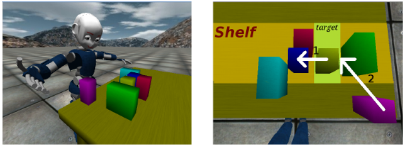
Outline

Probabilistic Logic Programs (ProbLog)

"Smokers" (Domingos et al. 2008; Fierens et al. 2015)

```
0.2::stress(P) :- person(P).
0.3::influences(P1, P2) :- friend(P1, P2).
0.1::cancer_spont(P) :- person(P).
0.3::cancer_smoke(P) :- person(P).
     smokes(X) :- stress(X).
     smokes(X) :- smokes(Y), influences(Y, X).
     cancer(P) :- cancer_spont(P).
     cancer(P) :- smokes(P), cancer_smoke(P).
     person(mary).
     person(albert).
     friend(albert, mary).
```

Applications



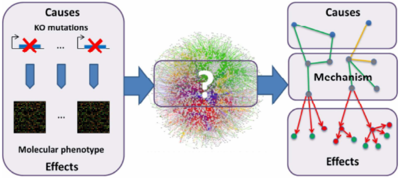
Moldovan et al. 2012

Q1: In a group of 10 people, 60 percent have brown eyes. Two people are to be selected at random from the group. What is the probability that neither person selected will have brown eyes?
Q2: Mike has a bag of marbles with 4 white, 8 blue, and 6 red marbles. He pulls out one marble from the bag and it is red. What is the probability that the second marble he pulls out of the bag is white?

Dries et al. 2017

```
is_malignant(Case):-
        biopsyProcedure(Case,usCore),
        changes_Sizeinc(Case,missing),
        feature_shape(Case).
is_malignant(Case):-
        assoFinding(Case,asymmetry),
        breastDensity(Case,scatteredFDensities),
        vacuumAssisted(Case,yes).
is_malignant(Case):-
        needleGauge(Case,9),
        offset(Case,14),
        vacuumAssisted(Case,yes).
```

Côrte-Real, Dutra, and Rocha 2017



De Maeyer et al. 2013

Probabilistic Inference: Reasoning by Hand

Let $a \oplus b := a + b - ab$. Then

$$\begin{aligned}\Pr[\texttt{cancer}(\mathit{mary})] = {} & \Pr[\texttt{cancer\_spont}(\mathit{mary})] \\ & \oplus \Pr[\texttt{smokes}(\mathit{mary})] \\ & \times \Pr[\texttt{cancer\_smoke}(\mathit{mary})]\end{aligned}$$

```
cancer(P) :- cancer_spont(P).
cancer(P) :- smokes(P), cancer_smoke(P).
```

Probabilistic Inference: Reasoning by Hand

Let $a \oplus b := a + b - ab$. Then
$\Pr[\texttt{cancer}(mary)] = 0.1 \oplus 0.3 \times \Pr[\texttt{smokes}(mary)]$

```
0.1::cancer_spont(P) :- person(P).
0.3::cancer_smoke(P) :- person(P).
```

Probabilistic Inference: Reasoning by Hand

Let $a \oplus b := a + b - ab$. Then

$$\begin{aligned}
\Pr[\texttt{cancer}(mary)] &= 0.1 \oplus 0.3 \times \Pr[\texttt{smokes}(mary)] \\
\Pr[\texttt{smokes}(mary)] &= \Pr[\texttt{stress}(mary)] \\
&\oplus \Pr[\texttt{smokes}(albert)] \\
&\times \Pr[\texttt{influences}(albert, mary)]
\end{aligned}$$

```
smokes(X) :- stress(X).
smokes(X) :- smokes(Y), influences(Y, X).
```

Probabilistic Inference: Reasoning by Hand

Let $a \oplus b \coloneqq a + b - ab$. Then

$\Pr[\texttt{cancer}(mary)] = 0.1 \oplus 0.3 \times \Pr[\texttt{smokes}(mary)]$

$\Pr[\texttt{smokes}(mary)] = 0.2 \oplus 0.3 \times \Pr[\texttt{smokes}(albert)]$

$$\boxed{\begin{array}{l} 0.2::\texttt{stress}(P) :- \texttt{person}(P). \\ 0.3::\texttt{influences}(P_1, P_2) :- \texttt{friend}(P_1, P_2). \end{array}}$$

Probabilistic Inference: Reasoning by Hand

Let $a \oplus b \coloneqq a + b - ab$. Then

$\Pr[\texttt{cancer}(\mathit{mary})] = 0.1 \oplus 0.3 \times \Pr[\texttt{smokes}(\mathit{mary})]$
$\Pr[\texttt{smokes}(\mathit{mary})] = 0.2 \oplus 0.3 \times \Pr[\texttt{smokes}(\mathit{albert})]$
$\Pr[\texttt{smokes}(\mathit{albert})] = \Pr[\texttt{stress}(\mathit{albert})] = 0.2$

```
0.2::stress(P) :- person(P).
     smokes(X) :- stress(X).
```

Probabilistic Inference: Probabilities of Worlds

```
0.2::stress(P) :- person(P).
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     smokes(X) :- stress(X).
     smokes(X) :- smokes(Y), influences(Y, X).
     cancer(P) :- cancer_spont(P).
     cancer(P) :- smokes(P), cancer_smoke(P).
     person(mary).
     person(albert).
     friend(albert, mary).
```

Probabilistic Inference: Probabilities of Worlds

$$\mathtt{cancer}(\mathit{mary}) = \top$$

$$\Pr(\mathit{world}) = 0.2 \times (1 - 0.3) \times 0.1 \times 0.3$$

$\cancel{0.2 ::}\,\mathtt{stress}(P) :\text{-}\ \mathtt{person}(P).$

$\cancel{0.3 :: \mathtt{influences}(P_1, P_2) :\text{-}\ \mathtt{friend}(P_1, P_2).}$

$\cancel{0.1 ::}\,\mathtt{cancer\_spont}(P) :\text{-}\ \mathtt{person}(P).$

$\cancel{0.3 ::}\,\mathtt{cancer\_smoke}(P) :\text{-}\ \mathtt{person}(P).$

$\mathtt{smokes}(X) :\text{-}\ \mathtt{stress}(X).$

$\cancel{\mathtt{smokes}(X) :\text{-}\ \mathtt{smokes}(Y), \mathtt{influences}(Y, X).}$

$\mathtt{cancer}(P) :\text{-}\ \mathtt{cancer\_spont}(P).$

$\mathtt{cancer}(P) :\text{-}\ \mathtt{smokes}(P), \mathtt{cancer\_smoke}(P).$

$\mathtt{person}(\mathit{mary}).$

$\mathtt{person}(\mathit{albert}).$

$\mathtt{friend}(\mathit{albert}, \mathit{mary}).$

Probabilistic Inference: Probabilities of Worlds

$$\texttt{cancer}(\mathit{mary}) = \top$$

$$\Pr(\mathit{world}) = 0.2 \times 0.3 \times 0.1 \times (1 - 0.3)$$

$\cancel{0.2 ::}\texttt{stress}(P) :- \texttt{person}(P).$

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$\cancel{0.1 ::}\texttt{cancer\_spont}(P) :- \texttt{person}(P).$

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$\texttt{smokes}(X) :- \texttt{stress}(X).$

$\texttt{smokes}(X) :- \texttt{smokes}(Y), \texttt{influences}(Y, X).$

$\texttt{cancer}(P) :- \texttt{cancer\_spont}(P).$

$\cancel{\texttt{cancer}(P) :- \texttt{smokes}(P), \texttt{cancer\_smoke}(P).}$

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$\texttt{person}(\mathit{albert}).$

$\texttt{friend}(\mathit{albert}, \mathit{mary}).$

Probabilistic Inference: Probabilities of Worlds

$$\texttt{cancer}(\mathit{mary}) = \top$$

$$\Pr(\mathit{world}) = (1 - 0.2) \times (1 - 0.3) \times 0.1 \times 0.3$$

$\cancel{0.2 :: \texttt{stress}(P) :- \texttt{person}(P).}$

$\cancel{0.3 :: \texttt{influences}(P_1, P_2) :- \texttt{friend}(P_1, P_2).}$

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$\cancel{\texttt{cancer}(P) :- \texttt{smokes}(P), \texttt{cancer\_smoke}(P).}$

$\texttt{person}(\mathit{mary}).$

$\texttt{person}(\mathit{albert}).$

$\texttt{friend}(\mathit{albert}, \mathit{mary}).$

Probabilistic Inference: Probabilities of Worlds

$$\texttt{cancer}(\mathit{mary}) = \bot$$

$$\Pr(\mathit{world}) = (1 - 0.2) \times (1 - 0.3) \times (1 - 0.1) \times (1 - 0.3)$$

$\cancel{0.2 :: \texttt{stress}(P) :- \texttt{person}(P).}$

$\cancel{0.3 :: \texttt{influences}(P_1, P_2) :- \texttt{friend}(P_1, P_2).}$

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$\cancel{\texttt{smokes}(X) :- \texttt{stress}(X).}$

$\cancel{\texttt{smokes}(X) :- \texttt{smokes}(Y), \texttt{influences}(Y, X).}$

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$\texttt{person}(\mathit{mary}).$

$\texttt{person}(\mathit{albert}).$

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Inference Algorithms and Knowledge Compilation Maps

NNF	negation normal form
BDD	binary decision diagrams
SDD	sentential decision diagrams
k-Best	only use the *k* most probable proofs
d-DNNF	deterministic decomposable negation normal form

Inference Algorithms and Knowledge Compilation Maps

- **NNF** negation normal form
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- ***k*-Best** only use the k most probable proofs
- **d-DNNF** deterministic decomposable negation normal form
  - for every pair $\alpha \vee \beta$, we have $\alpha \wedge \beta = \bot$
  - for every pair $\alpha \wedge \beta$, no atoms are shared between α and β

Inference Algorithms and Knowledge Compilation Maps

NNF negation normal form

BDD binary decision diagrams

SDD sentential decision diagrams

k-Best only use the *k* most probable proofs

d-DNNF deterministic decomposable negation normal form

- for every pair $\alpha \vee \beta$, we have $\alpha \wedge \beta = \perp$
- for every pair $\alpha \wedge \beta$, no atoms are shared between α and β
- examples:

  ✗✗ $(A \vee C) \wedge (A \vee \neg B)$

Inference Algorithms and Knowledge Compilation Maps

NNF negation normal form

BDD binary decision diagrams

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***k*-Best** only use the k most probable proofs

d-DNNF deterministic decomposable negation normal form

- for every pair $\alpha \lor \beta$, we have $\alpha \land \beta = \bot$
- for every pair $\alpha \land \beta$, no atoms are shared between α and β
- examples:
  - ✗✗ $(A \lor C) \land (A \lor \neg B)$
  - ✗✓ $C \land (A \lor \neg B)$

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- examples:
  - ✗✗ $(A \lor C) \land (A \lor \neg B)$
  - ✗✓ $C \land (A \lor \neg B)$
  - ✓✗ $B \land C \land [(B \land A) \lor \neg B]$

Inference Algorithms and Knowledge Compilation Maps

- NNF negation normal form
- BDD binary decision diagrams
- SDD sentential decision diagrams
- k-Best only use the k most probable proofs
- d-DNNF deterministic decomposable negation normal form
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  - examples:

    ✗✗ $(A \vee C) \wedge (A \vee \neg B)$

    ✗✓ $C \wedge (A \vee \neg B)$

    ✓✗ $B \wedge C \wedge [(B \wedge A) \vee \neg B]$

    ✓✓ $C \wedge [(B \wedge A) \vee \neg B]$

Example Diagrams for $C \wedge (A \vee \neg B)$

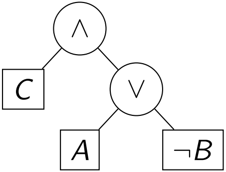

Figure: NNF

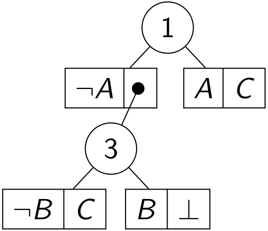

Figure: SDD

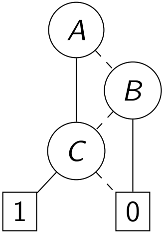

Figure: BDD

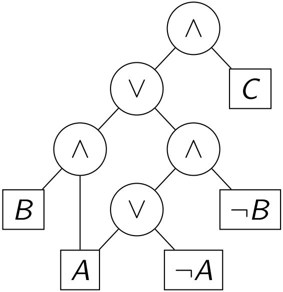

Figure: d-DNNF

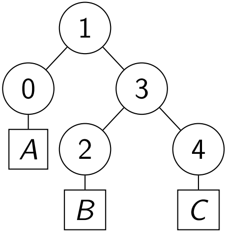

Figure: vtree

What Characterises a (Probabilistic) Logic Program?

$0.2 :: \texttt{stress}(\mathsf{P}) \texttt{:-} \texttt{person}(\mathsf{P}).$

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- predicates, arities

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$\texttt{smokes}(X) \text{ :- } \texttt{stress}(X).$

$\texttt{smokes}(X) \text{ :- } \texttt{smokes}(Y), \texttt{influences}(Y, X).$

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- predicates, arities
- variables

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- predicates, arities
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- predicates, arities
- variables
- constants
- probabilities
- length

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```
0.2::stress(P) :- person(P).
0.3::influences(P1, P2) :- friend(P1, P2).
0.1::cancer_spont(P) :- person(P).
0.3::cancer_smoke(P) :- person(P).
     smokes(X) :- stress(X).
     smokes(X) :- smokes(Y), influences(Y, X).
     cancer(P) :- cancer_spont(P).
     cancer(P) :- smokes(P), cancer_smoke(P).
     person(mary).
     person(albert).
     friend(albert, mary).
```

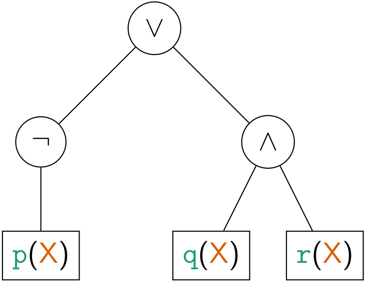
- predicates, arities
- variables
- constants
- probabilities
- length
- complexity

Formulas As Trees

$\neg \mathtt{p}(X) \lor (\mathtt{q}(X) \land \mathtt{r}(X))$

Formulas As Trees

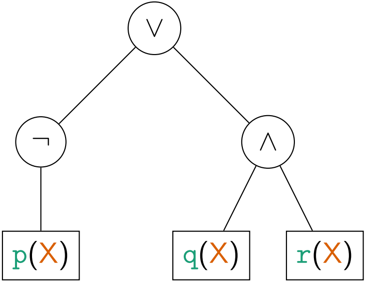
$\neg \mathtt{p}(X) \vee (\mathtt{q}(X) \wedge \mathtt{r}(X))$



Formulas As Trees

$\neg p(X) \vee (q(X) \wedge r(X))$

s:	0	0	0	1	2	2	6
v:	$\vee$	$\neg$	$\wedge$	p(X)	q(X)	r(X)	$\top$



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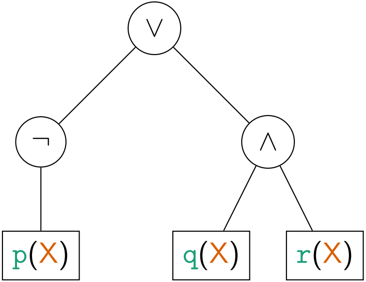

- s is a forest with $T = 2$ trees

Formulas As Trees

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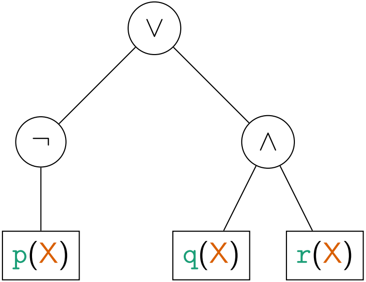

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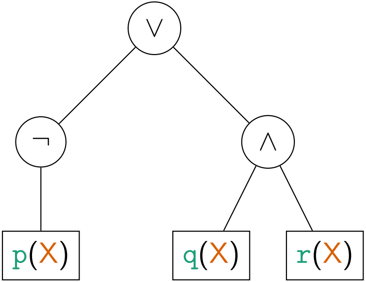


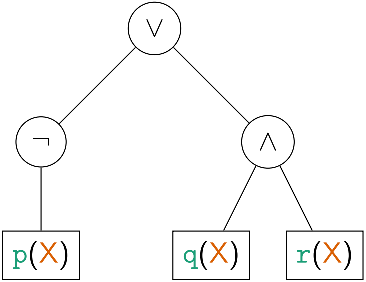
- s is a forest with $T = 2$ trees

- s is sorted
- $s_i \neq i \implies v_i \neq \top$

Formulas As Trees

$\neg p(X) \lor (q(X) \land r(X))$

s:	0	0	0	1	2	2	6
v:	$\lor$	$\neg$	$\land$	p(X)	q(X)	r(X)	$\top$



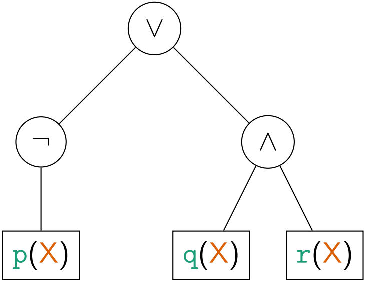
- s is a forest with $T = 2$ trees
- length $L = 7$
- number of nodes $N := L - T + 1 = 6$
- for $i = 1, \dots, L - 1$,
  - if $i < N$, then $s_i < i$
  - else $s_i = i$ and $v_i = \top$

- s is sorted
- $s_i \neq i \implies v_i \neq \top$

Formulas As Trees

$\neg \mathtt{p}(X) \lor (\mathtt{q}(X) \land \mathtt{r}(X))$

s:	0	0	0	1	2	2	6
v:	$\lor$	$\neg$	$\land$	p(X)	q(X)	r(X)	$\top$
c:	2	1	2	0	0	0	0



- s is a forest with $T = 2$ trees
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- for $i = 1, \ldots, L - 1$,
  - if $i < N$, then $s_i < i$
  - else $s_i = i$ and $v_i = \top$
- $c_i = 0 \iff v_i = \top$ or an atom
- $c_i = 1 \iff v_i = \neg$
- $c_i > 1 \iff v_i \in \{\land, \lor\}$

- s is sorted
- $s_i \neq i \implies v_i \neq \top$

Variable Symmetry Breaking

Let $\{W, X, Y\}$ be the set of variables. Then

$$\texttt{smokes}(X) :\!-\, \texttt{smokes}(Y), \texttt{influences}(Y, X)$$

is equivalent to

$$\texttt{smokes}(Y) :\!-\, \texttt{smokes}(X), \texttt{influences}(X, Y)$$

and to

$$\texttt{smokes}(W) :\!-\, \texttt{smokes}(X), \texttt{influences}(X, W)$$

Variable Symmetry Breaking

X	Y	Y	X

Occurrences

(channeling)

$W \mapsto \emptyset$

$X \mapsto \{0, 3\}$

$Y \mapsto \{1, 2\}$

Introductions

$1 + \min \mathrm{occurrences}(v)$ or 0

$W \mapsto 0$

$X \mapsto 1$

$Y \mapsto 2$

sorted!

Variable Symmetry Breaking

Y	X	X	Y

Occurrences

(channeling)

W $\mapsto \emptyset$

X $\mapsto \{1, 2\}$

Y $\mapsto \{0, 3\}$

Introductions

$1 + \min \text{occurrences}(v)$ or 0

W $\mapsto 0$

X $\mapsto 2$

Y $\mapsto 1$

not sorted!

Variable Symmetry Breaking

W	X	X	W

Occurrences

(channeling)

$W \mapsto \{0, 3\}$

$X \mapsto \{1, 2\}$

$Y \mapsto \emptyset$

Introductions

$1 + \min \text{occurrences}(v)$ or 0

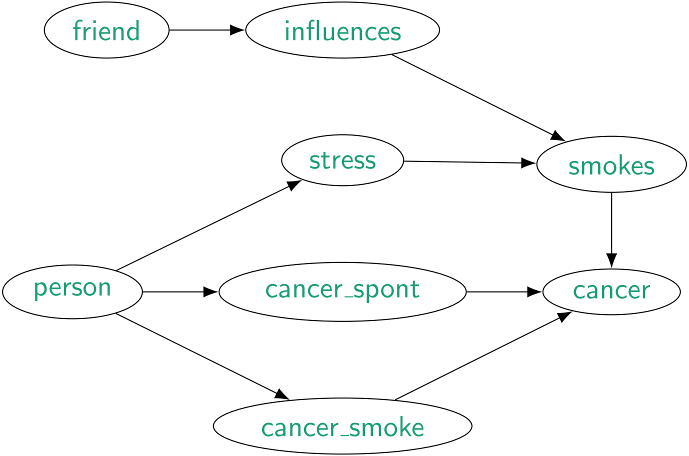
$W \mapsto 1$

$X \mapsto 2$

$Y \mapsto 0$

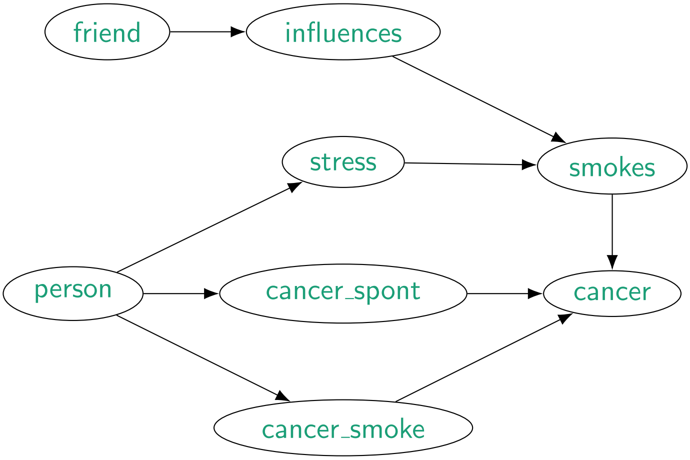
not sorted!

Predicate Dependency Graph



Stratification and Negative Cycles

$0.1 :: \texttt{friend}(X, Y) \texttt{:-} \backslash + \texttt{smokes}(Y).$



Stratification and Negative Cycles

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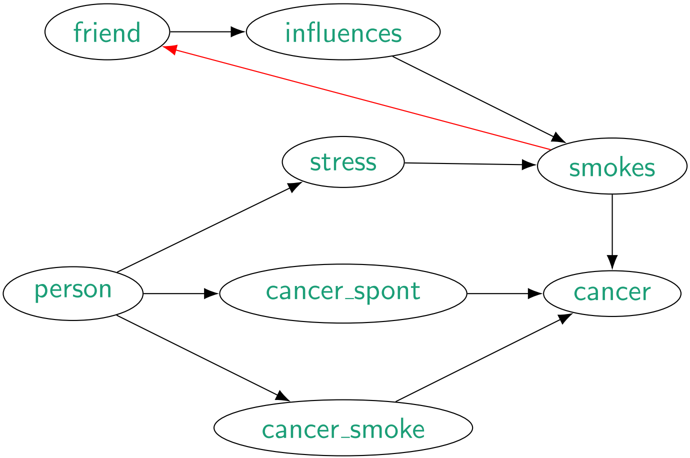
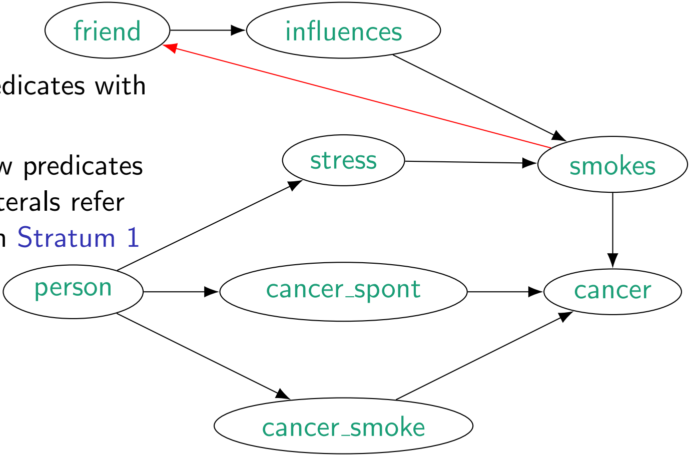


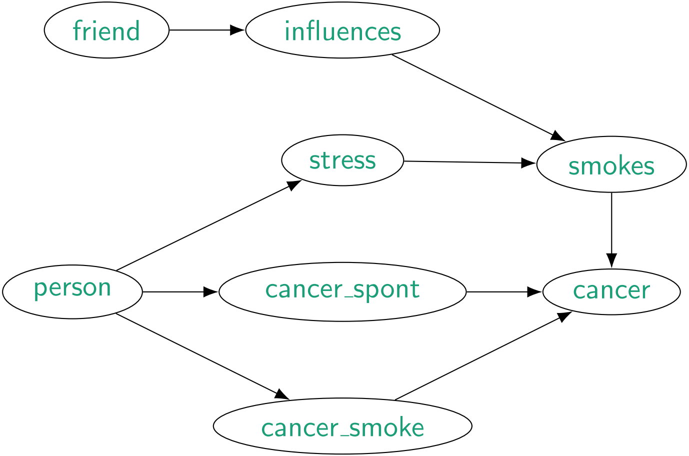
Stratification and Negative Cycles

$0.1 :: \texttt{friend}(X, Y) \text{ :- } \backslash+ \texttt{smokes}(Y).$



- Stratum 1: predicates with no negation
- Stratum 2: new predicates s.t. negative literals refer to predicates in Stratum 1

Predicate Independence: $\mathrm{friend} \perp\!\!\!\perp \mathrm{stress}$



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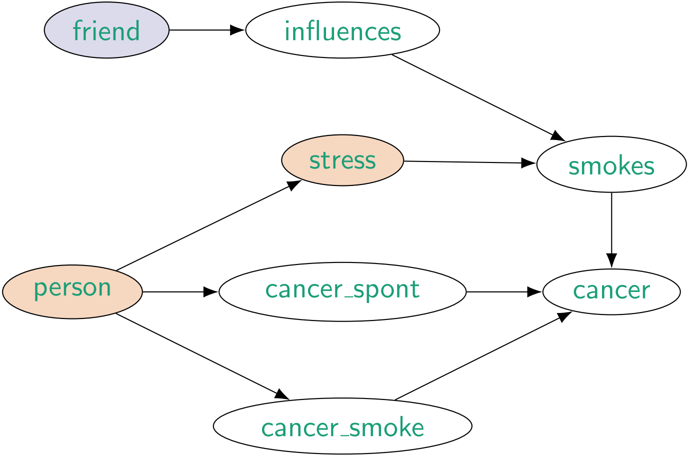
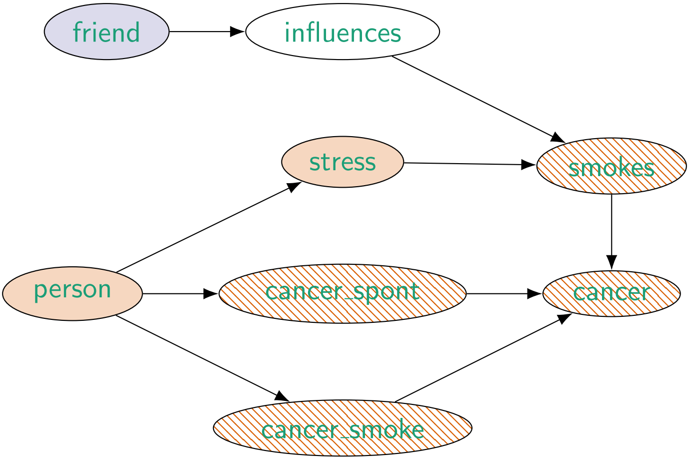
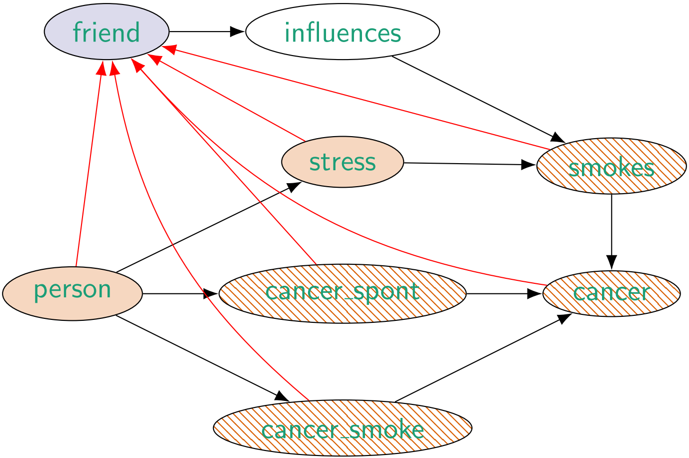


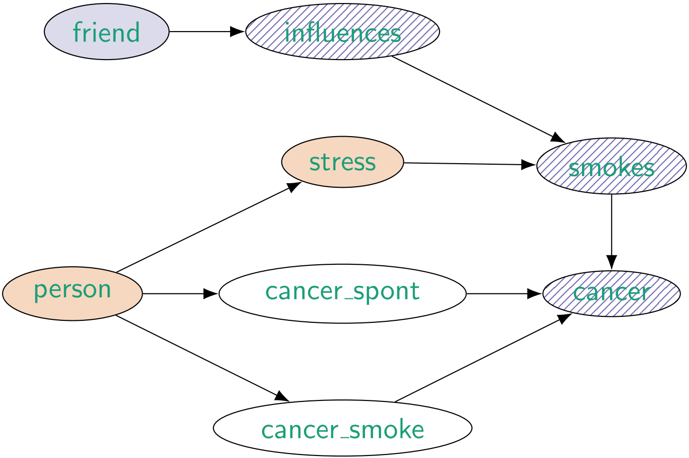
Predicate Independence: $\mathrm{friend} \perp\!\!\!\perp \mathrm{stress}$



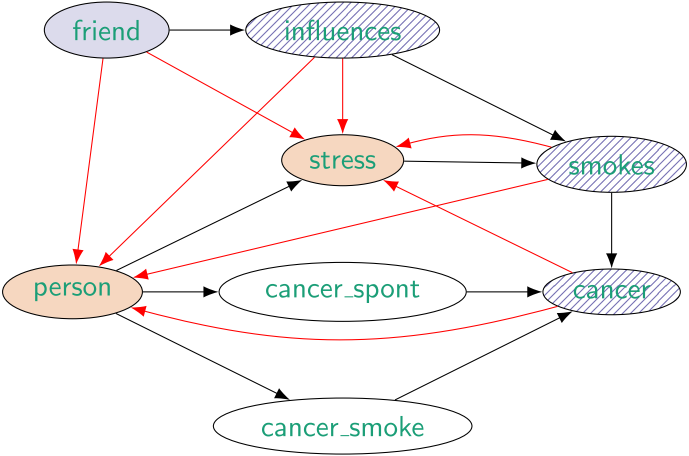
Predicate Independence: $\text{friend} \perp\!\!\!\perp \text{stress}$



Predicate Independence: friend $\perp\!\!\!\perp$ stress



Predicate Independence: $\text{friend} \perp\!\!\!\perp \text{stress}$



One-Liners

Setup

- predicate p/1
- variable X
- no constants
- 1 clause
- 4 nodes
- no negative cycles

(All) Programs

- p(X).
- 0.7 :: p(X) :- p(X).
- 0.8 :: p(X) :- p(X); p(X).
- 0.7 :: p(X) :- p(X), p(X).
- 0.1 :: p(X) :- p(X); p(X); p(X).
- 0.8 :: p(X) :- p(X), p(X), p(X).

Symmetry Breaking in Action

Setup

- predicate `p`/3
- variables: X, Y, Z
- no constants
- 1 clause
- 1 node
- no cycles at all

(All) Programs

- 0.8 :: p(Z, Z, Z).
- p(Y, Y, Z).
- p(Y, Z, Y).
- p(Y, Z, Z).
- 0.1 :: p(X, Y, Z).

A Larger Example

Setup

- predicates: `p`/1, `q`/2, `r`/3
- variables: X, Y, Z
- constants: a, b, c
- 5 clauses
- 5 nodes
- no negative cycles

A Random Program

$$
\begin{aligned}
&\mathtt{p}(b) \text{ :- } \backslash\!+ (\mathtt{q}(a, b), \mathtt{q}(\mathsf{X}, \mathsf{Y}), \mathtt{q}(\mathsf{Z}, \mathsf{X})).\\
0.4 :: &\mathtt{q}(\mathsf{X}, \mathsf{X}) \text{ :- } \backslash\!+ \mathtt{r}(\mathsf{Y}, \mathsf{Z}, a).\\
&\mathtt{q}(\mathsf{X}, a) \text{ :- } \mathtt{r}(\mathsf{Y}, \mathsf{Y}, \mathsf{Z}).\\
&\mathtt{q}(\mathsf{X}, a) \text{ :- } \mathtt{r}(\mathsf{Y}, b, \mathsf{Z}).\\
&\mathtt{r}(\mathsf{Y}, b, \mathsf{Z}).
\end{aligned}
$$

Examples of Predicate Independence

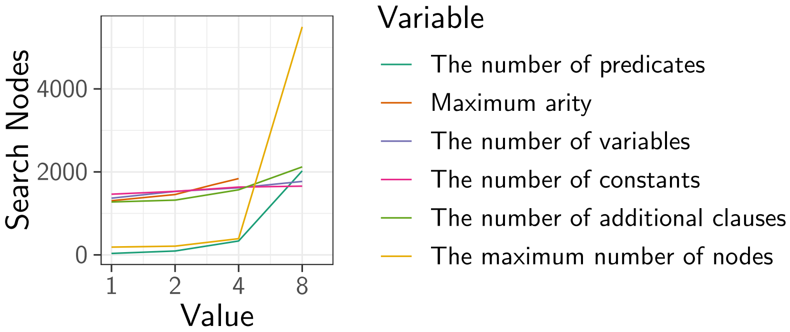
Setup

- predicates: $\mathtt{p}/1$, $\mathtt{q}/1$, $\mathtt{r}/1$
- no variables
- constant a
- 3 clauses
- 3 nodes
- no negative cycles
- $\mathtt{p} \perp\!\!\!\perp \mathtt{q}$

A Few Random Programs

- $0.5 :: \mathtt{p}(a) :- \mathtt{p}(a); \mathtt{p}(a).$
  $0.2 :: \mathtt{q}(a) :- \mathtt{q}(a), \mathtt{q}(a).$
  $0.4 :: \mathtt{r}(a) :- \backslash + \mathtt{q}(a).$
- $\mathtt{p}(a) :- \mathtt{p}(a).$
  $0.5 :: \mathtt{q}(a) :- \mathtt{r}(a); \mathtt{q}(a).$
  $\mathtt{r}(a) :- \mathtt{r}(a); \mathtt{r}(a).$
- $\mathtt{p}(a) :- \mathtt{p}(a); \mathtt{p}(a).$
  $0.6 :: \mathtt{q}(a) :- \mathtt{q}(a).$
  $0.7 :: \mathtt{r}(a) :- \backslash + \mathtt{q}(a).$

Scalability



What Programs Should We Generate?

- each program is divided into:
  - rules
    - e.g., `0.2::stress(P) :- person(P).`
  - facts
    - e.g., `friend(albert, mary).`
- predicates, variables, nodes: 2, 4, 8
- maximum arity: 1, 2, 3
- all possible numbers of pairs of independent predicates
- 10 programs per configuration
  - fully restarting the constraint solver
- probabilities sampled from $\{0.1, 0.2, \ldots, 0.9\}$
- query: random unlisted fact

Rules

```
0.2::stress(P) :- person(P).
```

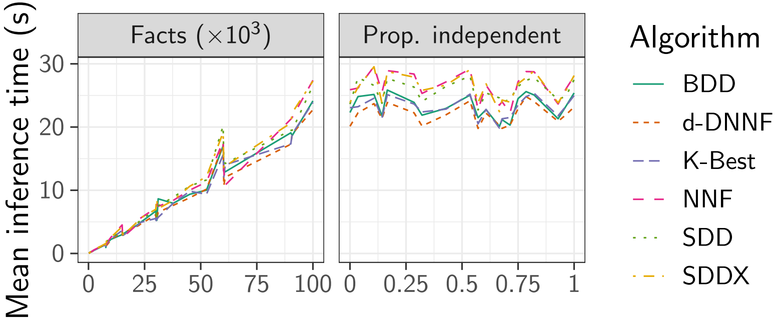
- no constants, no empty bodies
- one rule per predicate
- all rules are probabilistic

Facts

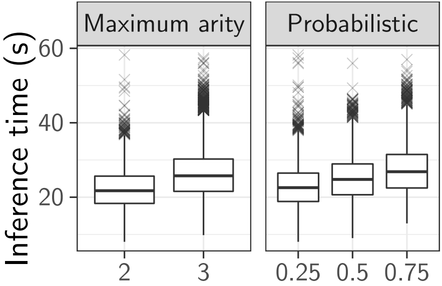
`friend`(*albert*, *mary*).

- proportion probabilistic: 25%, 50%, 75%
- constants: 100, 200, 400
- number of facts: 10^3, 10^4, 10^5
    - but only up to 75% of all possible facts

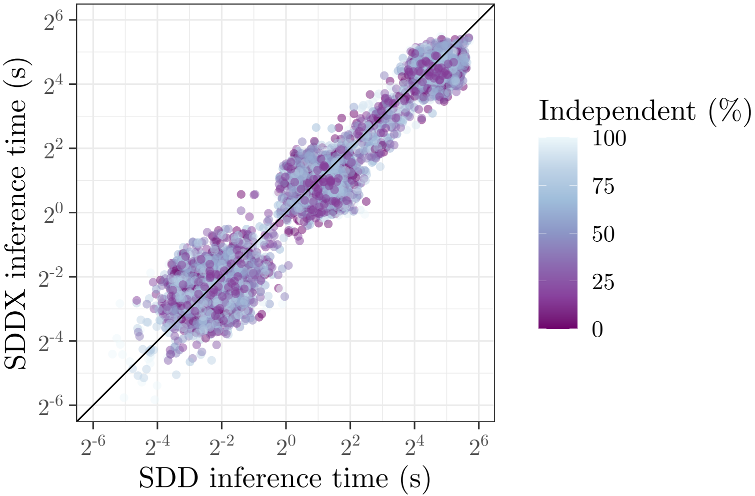
Properties of Programs vs. Inference Algorithms



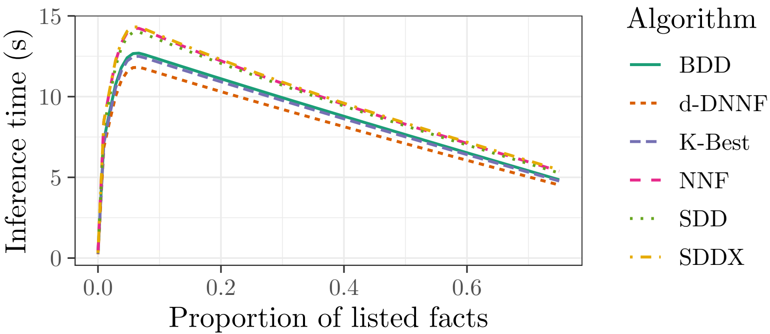
Properties of Programs vs. Inference Algorithms



How Encodings Compare Across Instances



The Ratio of Listed Facts to Possible Facts



Summary

- The model can generate (approximately) realistic instances of reasonable size
- The main performance bottleneck can be addressed by generating programs with a simpler structure
- Open questions and future work
  - Can we ensure uniform sampling?
  - Why do all of the algorithms behave so similarly?
  - Why does independence have no effect on inference time?